

3. RLC circuit analysis

This video completes the youtube video and its aim is not to substitute the video.

3.1. Introduction

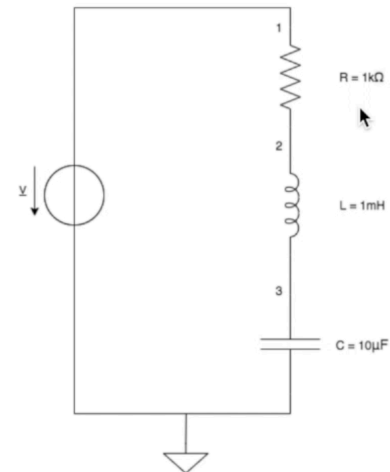
By means of LTSpice, we will observe the impedance of an RLC circuit in the frequency domain and in the time domain.

3.2. RLC circuit

RLC Circuit in series

The impedance of the circuit is : $\underline{Z} = R + j(\omega L - \frac{1}{\omega C})$

The amplitude of the current is : $I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

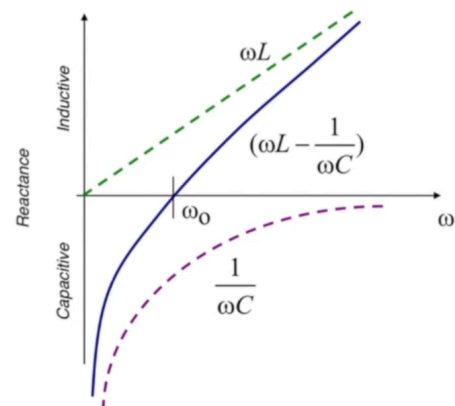


Resonance

For $\omega = \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \approx 1600$ [Hz]

The impedance of the circuit is resistive : $\underline{Z} = R$

The amplitude of the current is maximal : $I = \frac{V}{R}$

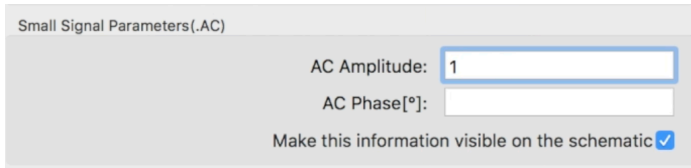


RLC Circuit in series and resonance

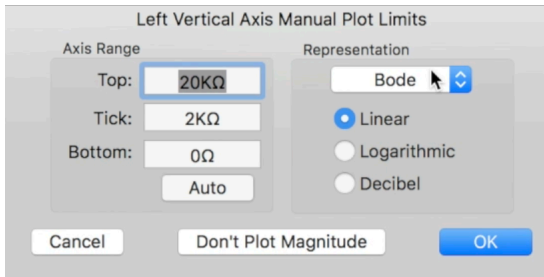
Here are the RLC circuit, its impedance and its current amplitude. In the first simulation, we will study the resonance frequency f_0 of this circuit. It is the frequency for which the reactance of the impedance is zero. At this frequency, the impedance is resistive and the amplitude of the current maximal. If the frequency is smaller than f_0 , the reactance is negative so capacitive. If the frequency is greater than f_0 , the reactance is positive so inductive.

3.3. .ac - impedance between two frequencies

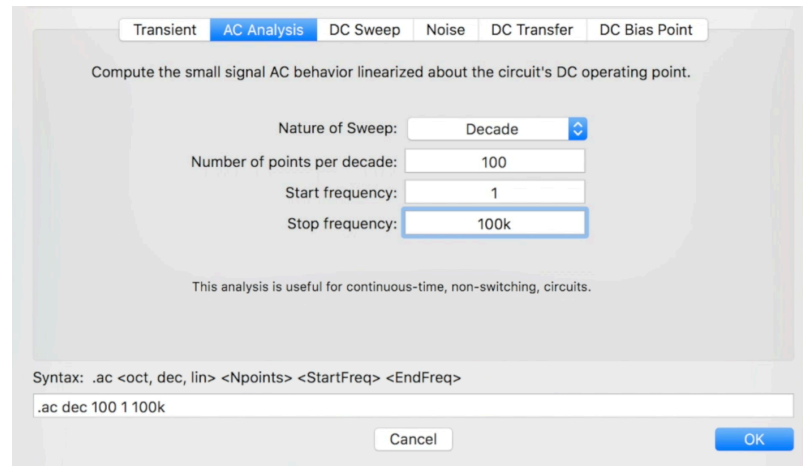
To do this first simulation, we configure the small signals parameters of the voltage source with an AC amplitude of 1V. Then, we configure the directive AC analysis (.ac) as on the picture. This directive is used to do a frequency sweep between two



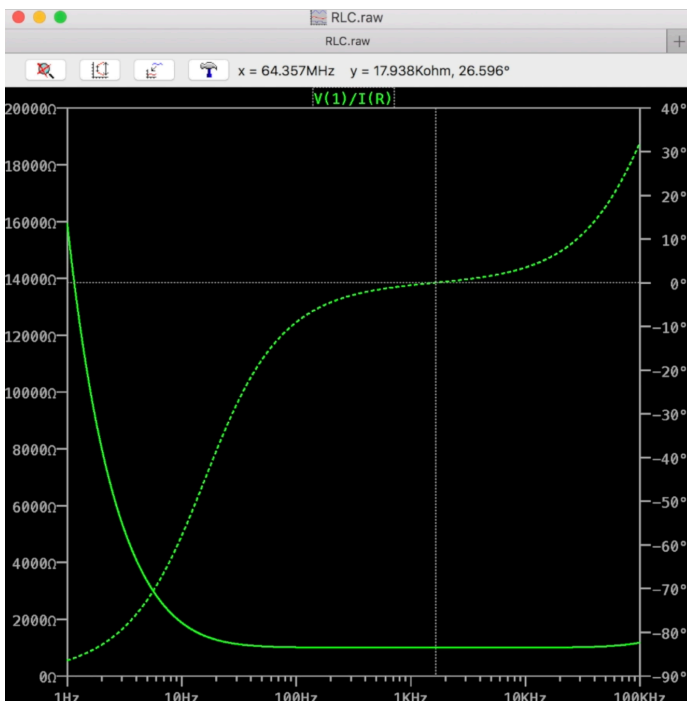
Voltage source : small signal parameters (.AC)



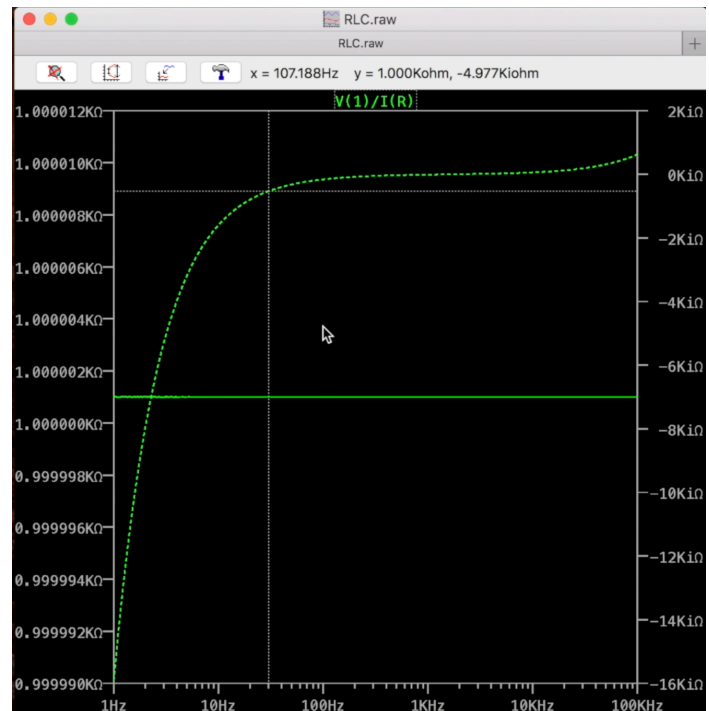
Plot pane : to change the representation



Directive : AC analysis (.ac)



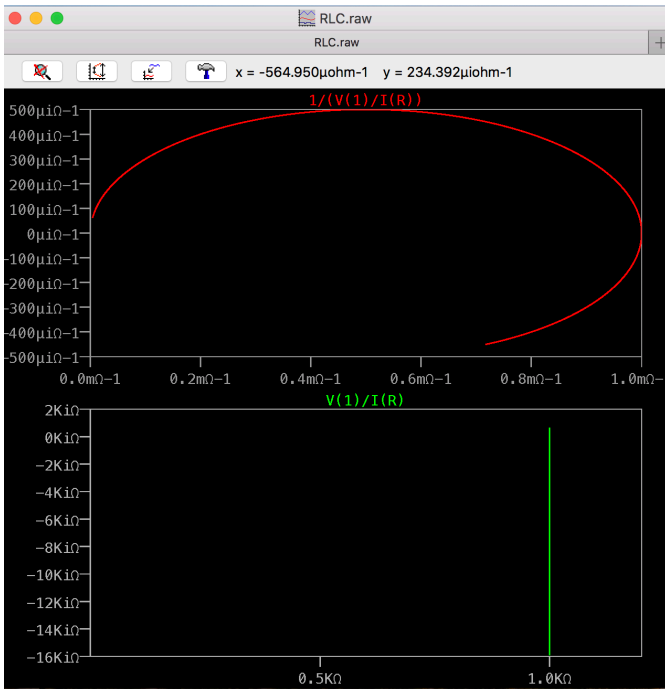
Impedance : Bode representation



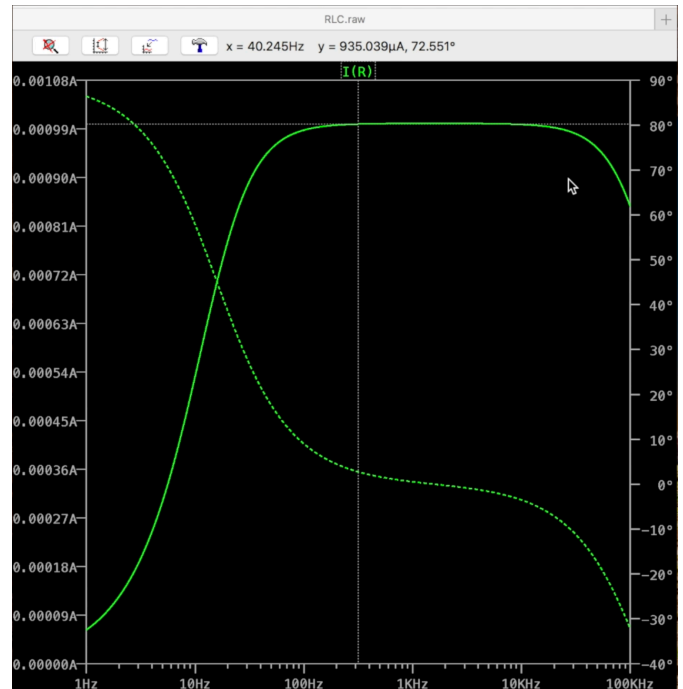
Impedance : Cartesian representation

limits. We obtain a plot pane on which we display the impedance in the Bode representation. The x-axis is the frequency. The right axis corresponds to the solid line, the magnitude of the impedance and the left one to the dash line, the phase of the impedance. We can then observe the discussed points in the theory.

When you right click on the left axis, you can change the representation. In the Bode representation, you can also choose a linear, logarithmic or a decibel scale. With the Cartesian representation, we have the real part of the expression on the left axis and the imaginary part on the right one. The x-axis is still the frequency. The Nyquist representation corresponds to the complex plane with the real part on the x-axis and the imaginary part on the y-axis.



Impedance (green) and admittance (red) :
Nyquist representation



Current : Bode representation

In the Nyquist representation, the impedance is a line, as the resistance is constant ($=R$). We display also the admittance, the inverse of the impedance. We obtain a kind of circle because the inverse of a line in the complex plane is a circle.

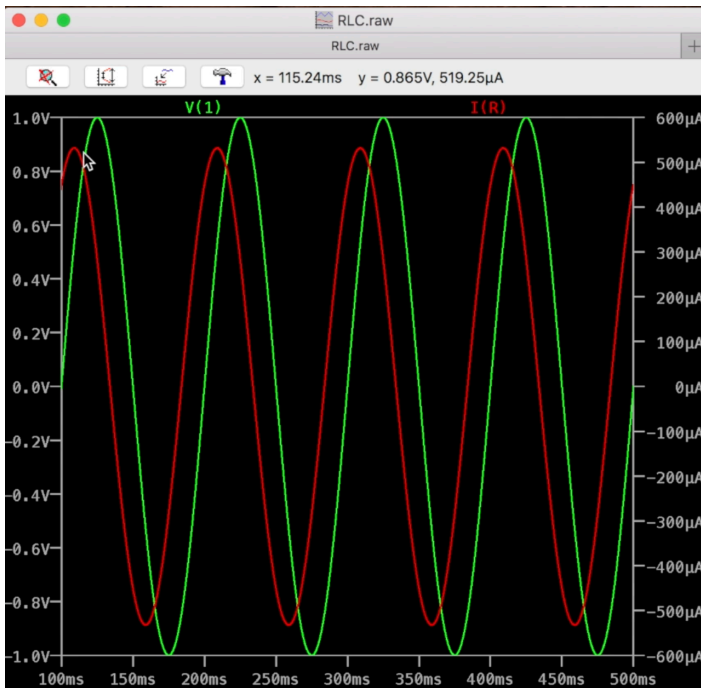
Finally, we display the current in the Bode representation. As studied in the theory, we have a maximum of the current amplitude at the resonance frequency and

$$\underline{I} = \frac{\underline{V}}{\underline{R}}.$$

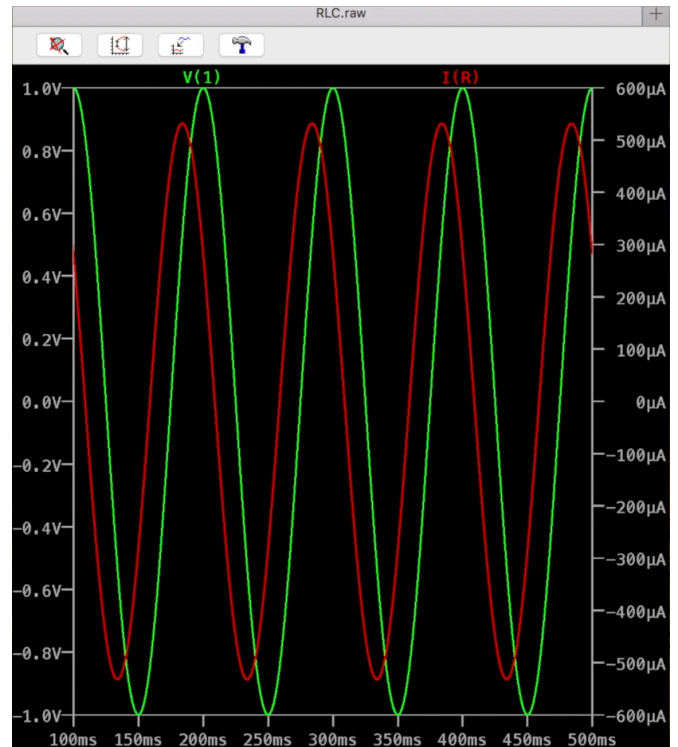
3.4. .tran - impedance at a frequency of 10Hz

We configure the voltage source with a sinusoidal signal of 1V and 10Hz. Then, we add the directive `.tran 0.5` to analyse the circuit from 0s to 0.5s. When we display the plot pane, we select 100ms for the left limit of the x-axis. Thus, we hide the transient state, which we are here not interested in. We display then the current and the voltage $V(1)$. At 10Hz, the current is head of the voltage because in the previous simulation, we have seen that the phase of the current at 10Hz was negative. We have : $\arg(\underline{I}) = \arg(\underline{V}(1)) - \arg(\underline{Z}) \Rightarrow \arg(\underline{I}) > \arg(\underline{V}(1))$.

Notice that in an exercise, when only the amplitude of the voltage applied on the circuit is imposed, you have the choice of its phase. The phase of the others phenomenons will then follows this choice. To understand this idea, we do two simulations with a different ϕ of the voltage source (see the last pictures).



Current (red) and voltage $V(1)$ (green) at a frequency of 10Hz with $\phi = 0^\circ$



Current (red) and voltage $V(1)$ (green) at a frequency of 10Hz with $\phi = 90^\circ$